Global Exponential Stability of Cellular Neural Network with Mixed Discrete and Distributed Delays

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Abstract: This paper is concerned with analysis problem for the global exponential stability of recurrent neural networks (RNNs) with mixed discrete and distributed delays, unlike other papers, the nodes are associated with the topology of network. By using Lyapunov-Krasovskii functional and Young inequality, we give the sufficient condition of global exponential stability of the new cellular neural network, in addition, the example is provided to illustrate the applicability of the result.

Keywords: topology; Lyapunov-Krasovskii function; Young inequality; global exponential stability

1. Introduction

In 1992, Chua and Roska introduced a delay cellular neural network (CNN). Delay cellular neural network has been applied in many fields nowadays, such as moving target detection, identification and classification, further, along with the rapid development of biological information in cell simulation; image analysis has been widely used in recent years. We must first concern the stability of system, because the main function of CNN is about to change an input image into a corresponding output image. The various generalizations of neural networks have attracted attention of the scientific community due to their promising potentiality for tasks of classification, associative memory, parallel computation and the ability to difficult optimization [1-5]. Such applications rely on the existence of equilibrium points and the qualitative properties of neural networks. The time delay is commonly existed in various engineering systems such as chemical processes, hydraulic and rolling mill systems, etc. These effects are unavoidably existed in the implementation of neural networks, and may cause undesirable dynamic network behaviors such as oscillation and instability. Therefore, it is important to investigate the stability of delayed neural networks. The stability analysis of neural networks plays an important role in the designs and applications. A large number of the criteria on the stability of neural networks have been derived in the literature.

The global asymptotic stability results of different classes of delayed neural networks were proposed in [6-10, 22]. However, these results are only concerned with the asymptotic stability of networks without providing any conditions for exponential stability and any information about the decay rates of the delayed neural networks. Therefore, it is particularly important, when the exponential convergence rate is used to determine the speed of neural computations. Thus, it is important both theoretically and practically to determine the exponential stability and to estimate the exponential convergence rate for delayed neural networks. Considering this, the corresponding research results of many researchers have been reported in the literatures [11-15]. Neural network usually has a spatial nature due to the presence of various parallel pathways with a variety of axon sizes and lengths, so it is desirable to model them by introducing unbounded delays. Thus, there will be a distribution of conduction velocities along these pathways and a distribution of propagation delays. In these circumstances the signal propagation is not instantaneous and cannot be modeled with discrete delays and a more appropriate way is to incorporate continuously distributed delays in neural network models. In recent years there has been a growing research interest in study of neural networks with distributed delays. In fact, both discrete and distributed delays should be taken into account when modeling a realistic neural network [16-20].

Based on the above discussions, we consider a new mixed discrete and distributed delays cellular neural network described by a neutral integro-differential equation. The main purpose of this paper is to study the global exponential stability for neutral-type delayed neural networks. The structure of the neutral-type neural networks with distributed delays

under consideration is more general than the other papers existed in the literature. To the best of the author's knowledge, there were no global stability results for neutral-type neural networks. This paper is an attempt to this goal. By utilizing the Lyapunov-Krasovskii functional and Young inequality, we give the sufficient condition of global exponential stability of mixed discrete and distributed delays cellular neural network, In addition, the example is provided to illustrate the applicability of the result.

The paper is organized as follows: In Section 2, Problem formulation Model is stated and some definitions and lemmas are listed. Based on the Lyapunov stability theory and Young inequality, Main results and proofs about global exponential stability of multi-delay and distributed delay cellular neural network are listed in Section 3. In section 4, we give an example. We give the conclusion of this paper in Section 5.

2. Problem formulation Model

Consider the following multi-delay and distributed delay cellular neural network model：

$$\begin{cases} \dot{x}_i(t) = -d_i x_i(t) + \sum_{j=1}^{r} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{r} b_{ij} g_j(x_j(t-\tau_{ij}(t))) \\ \qquad + \sum_{j=1}^{r} c_{ij} \int_{-\infty}^{t} K_{ij}(t-s) h_j(x_j(s)) ds + I_i, \quad i = 1,2,\ldots,r \\ \dot{x}_i(t) = k_i(\bar{x}_r(t)) + \sum_{j=r+1}^{n} c_{ij} x_j(t), \ \ i = r+1, r+2, \ldots n, \\ x_i(\theta) = \phi_i(\theta), \ \rho = \max(\tau_{ij}(t)), -\rho \le \theta \le 0 \end{cases} \tag{1}$$

where $\phi_i(\theta)$ is bounded and continuous in the sub [0, +∞)， n is the number of the neurons in the neural network, , the constants a_{ij}, b_{ij} and c_{ij} denote, respectively, the connection weights, the discretely delayed connection weights and the distributively delayed connection weighted, of the jth neuron on the I neuron. $\bar{x}_r(t) = (x_1(t), x_2(t), \ldots, x_r(t))^T$ denotes the state of the ith neural neuron at time t, $f_j(x_j(t))$, $g_j(x_j(t))$ and $h_j(x_j(t))$ are the activation functions of the jth neuron at time t , I_i is the external bias on the ith neuron, d_i denotes the rate with which the ith neuron will reset its potential to the resting state in isolation when disconnected from the network and external inputs. $\tau_{ij} \ge 0$ is a bounded time-varying Delay，Kerner coefficient K_{ij}:[0,∞)→[0,∞) is continuous in the sub [0,∞) and satisfies $\int_0^{\infty} K_{ij}(s)ds = 1, \quad i, j = 1,2,\ldots,n$. $k_i(\cdot) \in C[R^r, R]$ and $k_i(0)=0$, $C=(c_{ij})_{(n-r)\times(n-r)}$ are real matrixes, which denote the strength of neuron interconnections.

In this paper, we make the following assumptions and definitions for the neuron activation functions.

Definition.1 x_i^* $(i = 1,2,\ldots,n)$ is the equilibrium point of (1) associated with a given I_i (i=1,2,…,n) is said to be globally exponentially stable, if there are positive constants k > 0and $\mu > 0$ such that every solution x_i^* (i=1,2,… n) of (1) satisfies as follows

$$\left|x_i(t) - x_i^*\right| \le \mu e^{-kt} \sup_{-\rho \le \theta \le 0} \left|\phi_i(\theta) - x_i^*\right|, \forall t > 0.$$

Definition.2 $\forall\, \phi(\theta) \in C\left([-\rho, 0], R^n\right)$, we definite

$|\phi| = \max\left\{\left\|\phi(\theta)\right\| : \theta \in [-\rho, 0]\right\}$, then we can get as follows

$$\left\|\phi - x^*\right\|_r^r = \sup_{-\infty \le \theta \le 0} \sum_{i=1}^{n} \left|\phi_i(\theta) - x_i^*\right|^r, r > 1$$

Assumption.1 (A1) For $i = 1,2,\ldots,n$, the neuron activation functions in (1) satisfy

$\left|f_j(s_1) - f_j(s_2)\right| \le \alpha_j^+ \left|s_1 - s_2\right|, \ \forall s_1 \ne s_2$

$$\left|g_j(s_1)-g_j(s_2)\right| \le \beta_j^+ \left|s_1-s_2\right|,\ \forall s_1 \ne s_2$$
$$\left|h_j(s_1)-h_j(s_2)\right| \le \gamma_j^+ \left|s_1-s_2\right|,\ \forall s_1 \ne s_2$$

where $\alpha_j^+, \beta_j^+, \gamma_j^+$ are constants.

Assumption.2 (A2)
The neuron activation functions $f_j(x_j(t))$, $g_j(x_j(t))$, $h_j(x_j(t))$ j =1,2,…,n are bounded.

Remark.1 The constants $\alpha_j^+, \beta_j^+, \gamma_j^+$ in Assumption 1 are allowed to be positive or zero. Thus, the resulting activation functions could be non-monotonic, and more general than the usual sigmoid functions.

Lemma.1 [21] (Rogers-Holder Inequality)

If $p>1, \frac{1}{p}+\frac{1}{q}=1,$ and $a_k>0, b_k>0\ (k=1,2,...,n)$, Then

$$\sum_{k=1}^{n} a_k b_k \le (\sum_{k=1}^{n} a_k^p)^{\frac{1}{p}} (\sum_{k=1}^{n} b_k^q)^{\frac{1}{q}}$$

Lemma.2 [22] (Young Inequality) if e > 0, h > 0, P > 1, $\frac{1}{P}+\frac{1}{q}=1,$ then we can get as follows

$$eh \le \frac{1}{p}e^p + \frac{1}{q}h^q = \frac{1}{p}e^p + \frac{p-1}{p}h^{\frac{p}{p-1}}$$

3. Main results and proofs

Theorm.1 f_j, g_j, h_j are Lipschitz continuous and $\dot{\tau}_{ij}(t)<0$, if there are constants
$\vartheta, \omega_i, q_{ij}, n_{ij}, h_{ij}, j_{ij}, l_{ij}, p_{ij}, q_{ji}, n_{ji}, h_{ji}, j_{ji}, l_{ji}, p_{ji}, \mu_1 \in R\ (i=1,2,...,n), \upsilon_{ij}=\upsilon_{ji}=\chi_{ij}=\chi_{ji}=1$
$(i, j=r+1,...,n), \omega_i>0, \varpi_i>0, \vartheta \ge 1$(when ϑ =1 =1, we must let
$q_{ij}=n_{ij}=h_{ij}=j_{ij}=l_{ij}=p_{ij}=q_{ji}=n_{ji}=h_{ji}=j_{ji}=l_{ji}=p_{ji}=1\ (i, j=1,2,...,r),$
$\upsilon_{ij}=\upsilon_{ji}=\chi_{ij}=\chi_{ji}=1\ (i, j=r+1,...,n)$

$$-\omega_i \vartheta d_i + (\vartheta-1)\sum_{j=1}^{r} \omega_i \left|a_{ij}\right|^{\frac{\vartheta-q_{ij}}{\vartheta-1}} \left|\alpha_j^+\right|^{\frac{\vartheta-n_{ij}}{\vartheta-1}} + \sum_{j=1}^{r} \omega_i \left|a_{ij}\right|^{q_{ij}} \left|\alpha_j^+\right|^{n_{ij}} + (\vartheta-1)\sum_{j=1}^{r} \omega_i \left|b_{ij}\right|^{\frac{\vartheta-h_{ij}}{\vartheta-1}} \left|\beta_j^+\right|^{\frac{\vartheta-j_{ij}}{\vartheta-1}}$$
$$+(\vartheta-1)\sum_{j=1}^{r} \omega_i \left|c_{ij}\right|^{\frac{\vartheta-l_{ij}}{\vartheta-1}} \left|\gamma_j^+\right|^{\frac{\vartheta-p_{ij}}{\vartheta-1}} + \sum_{j=1}^{r} \omega_i \left|c_{ij}\right|^{l_{ij}} \left|\gamma_j^+\right|^{p_{ij}} + \sum_{j=1}^{r} \omega_i \left|b_{ij}\right|^{h_{ij}} \left|\beta_j^+\right|^{j_{ij}} + \sum_{j=1}^{r} \omega_i \left|c_{ij}\right|^{l_{ij}} \left|\gamma_j^+\right|^{p_{ij}} \left|\beta_j^+\right|^{\vartheta} < 0, \quad (i=1,...,r)$$
$$\varpi_i \sum_{j=r+1}^{n} [(\vartheta-1)\left|k_i\right|^{\frac{\vartheta-\upsilon_{ij}}{\vartheta-1}} + \left|k_i\right|^{\upsilon_{ij}}]\ \mu_1 < 0, \varpi_i (n-r) \sum_{j=r+1}^{n} [(\vartheta-1)\left|c_{ij}\right|^{\frac{\vartheta-\chi_{ij}}{\vartheta-1}} + \left|c_{ij}\right|^{\chi_{ij}}] + \varepsilon(n-r) < 0 \quad (i=r+1,...,n)$$

Then, the equilibrium point of multi-delay and distributed delay cellular neural network x^* is global exponential stability.

Proof. We shift the equilibrium point $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ of (1) to the equation

$$u(t) = x(t) - x^* = [u_1(t), u_2(t), \ldots, u_n(t)]^T$$

Thus we can get as follows

$$\begin{cases} \dot{u}_i(t) = -d_i u_i(t) + \sum_{j=1}^{r} a_{ij} f_j^0(u_j(t)) + \sum_{j=1}^{r} b_{ij} g_j^0(u_j(t-\tau_{ij}(t))) \\ \qquad + \sum_{j=1}^{r} c_{ij} \int_{-\infty}^{t} K_{ij}(t-s) h_j^0(u_j(s)) ds, \quad i = 1, 2, ..., r \\ \dot{u}_i(t) = k_i(\bar{u}_r(t)) + \sum_{j=r+1}^{n} c_{ij} u_j(t), \ i = r+1, r+2, \ldots n, \end{cases} \tag{2}$$

where

$$\begin{aligned} f_j^0(u_j(t)) &= f_j(u_j(t) + x_j^*) - f_j(x_j^*), \\ g_j^0(u_j(t)) &= g_j(u_j(t) + x_j^*) - g_j(x_j^*), \\ h_j^0(u_j(t)) &= h_j(u_j(t) + x_j^*) - h_j(x_j^*), \\ \bar{u}_r(t) &= (u_1(t), u_2(t), \ldots, u_r(t))^T \end{aligned} \tag{3}$$

Consider multi-delay and distributed delay cellular neural networks associated with the problem of nonlinear equations

$$\begin{cases} d_i x_i^* = \sum_{j=1}^{r} a_{ij} f_j(x_i^*) + \sum_{j=1}^{r} b_{ij} g_j(x_i^*) + \sum_{j=1}^{r} c_{ij} \int_{-\infty}^{t} K_{ij}(t-s) h_j(x_i^*) ds + I_i, \ i = 1, 2, \ldots, r, \\ d_i x_i^* = k_i(\bar{x}_r^*(t)) + \sum_{j=r+1}^{n} c_{ij} x_j^*(t), \ i = r+1, r+2, \ldots n, \end{cases}$$

We design the following Lyapunov functional as follows

$$V(u,t) = \sum_{i=1}^{r} \omega_i \left\{ |u_i(t)|^{\vartheta} e^{\varepsilon t} + \sum_{j=1}^{r} |b_{ij}|^{h_{ij}} |\beta_j^+|^{j_{ij}} \int_{t-\tau_{ij}}^{t} |u_j(s)|^{\vartheta} e^{\varepsilon(s+\tau_{ij}(t))} ds + \sum_{i=r+1}^{n} \varpi_i \sum_{j=r+1}^{n} |u_i(t)|^{\vartheta} e^{\varepsilon t} \right.$$
$$\left. + \sum_{j=1}^{r} |c_{ij}|^{l_{ij}} |\gamma_j^+|^{p_{ij}} |\beta_j^+|^{\vartheta} \int_{0}^{t} |u_j(s-\tau_{ij})|^{\vartheta} e^{\varepsilon s} ds \right\} + \sum_{j=1}^{r} |c_{ij}|^{l_{ij}} |\gamma_j^+|^{p_{ij}} \int_{t-\tau_{ij}}^{t} |g_j(u_j(\xi))|^{\vartheta} e^{\varepsilon(\xi+\tau_{ij}(t))} d\xi$$

By (2), we calculate the Dini upper right derivative of the solution $V(u, t)$,

$$\begin{aligned} & D^+V(u,t) \\ &= \sum_{i=1}^{r} \omega_i \left\{ e^{\varepsilon t} [\varepsilon |u_i(t)|^{\vartheta} + \vartheta |u_i(t)|^{\vartheta-1} sign(u_i(t)) \dot{u}_i(t)] \right. \\ &\quad + e^{\varepsilon t} \left[\sum_{j=1}^{r} |b_{ij}|^{h_{ij}} |\beta_j^+|^{j_{ij}} |u_j(t)|^{\vartheta} e^{\varepsilon \tau_{ij}(t)} - \sum_{j=1}^{r} |b_{ij}|^{h_{ij}} |\beta_j^+|^{j_{ij}} |u_j(t-\tau_{ij})|^{\vartheta} (1 - \dot{\tau}_{ij}(t)) \right] \\ &\quad + e^{\varepsilon t} \left[\sum_{j=1}^{r} |c_{ij}|^{l_{ij}} |\gamma_j^+|^{p_{ij}} |g_j(u_j(t))|^{\vartheta} e^{\varepsilon \tau_{ij}(t)} - \sum_{j=1}^{n} |c_{ij}|^{l_{ij}} |\gamma_j^+|^{p_{ij}} |g_j(u_j(t-\tau_{ij}))|^{\vartheta} (1 - \dot{\tau}_{ij}(t)) \right] \\ &\quad \left. + \sum_{j=1}^{r} |c_{ij}|^{l_{ij}} |\gamma_j^+|^{p_{ij}} |\beta_j^+|^{\vartheta} e^{\varepsilon t} |u_j(t-\tau_{ij})|^{\vartheta} + \sum_{i=1}^{r} \sum_{j=1}^{r} |u_j(-\tau_{ij})|^{\vartheta} \dot{\tau}_{ij}(t) \right\} \\ &\quad + e^{\varepsilon t} \sum_{i=r+1}^{n} \varpi_i \left\{ \sum_{j=r+1}^{n} \vartheta |u_i(t)|^{\vartheta-1} sign(u_i(t)) \dot{u}_i(t) + \varepsilon \sum_{j=r+1}^{n} |u_i(t)|^{\vartheta} \right\} \end{aligned}$$

By $\dot{\tau}_{ij}(t) \le 0$, therefore we can get as follows

$$
\begin{aligned}
&D^{+}V(u,t)\\
&\le \sum_{i=1}^{r} \omega_i \Big\{ e^{\varepsilon t}[\varepsilon |u_i(t)|^{\vartheta} + \vartheta |u_i(t)|^{\vartheta-1} sign(u_i(t))\dot{u}_i(t)] + e^{\varepsilon t}\left[\sum_{j=1}^{r} |b_{ij}|^{h_{ij}} |\beta_j^{+}|^{j_{ij}} |u_j(t)|^{\vartheta} e^{\varepsilon\tau_{ij}(t)} - \sum_{j=1}^{r} |b_{ij}|^{h_{ij}} |\beta_j^{+}|^{j_{ij}} |u_j(t-\tau_{ij})|^{\vartheta} \right]\\
&+e^{\varepsilon t}\left[\sum_{j=1}^{r} |c_{ij}|^{l_{ij}} |\gamma_j^{+}|^{p_{ij}} |g_j(u_j(t))|^{\vartheta} e^{\varepsilon\tau_{ij}(t)} - \sum_{j=1}^{r} |c_{ij}|^{l_{ij}} |\gamma_j^{+}|^{p_{ij}} |g_j(u_j(t-\tau_{ij}))|^{\vartheta} \right] + \sum_{j=1}^{r} |c_{ij}|^{l_{ij}} |\gamma_j^{+}|^{p_{ij}} |\beta_j^{+}|^{\vartheta} e^{\varepsilon t} |u_j(t-\tau_{ij})|^{\vartheta} \Big\}\\
&+e^{\varepsilon t} \sum_{i=r+1}^{n} \varpi_i \left\{ \sum_{j=r+1}^{n} \vartheta |u_i(t)|^{\vartheta-1} sign(u_i(t)) \left(k_i(\bar{u}_r(t)) + \sum_{j=r+1}^{n} c_{ij} u_j(t) \right) + \varepsilon \sum_{j=r+1}^{n} |u_i(t)|^{\vartheta} \right\}\\
&\le \sum_{i=1}^{r} \omega_i \Big\{ e^{\varepsilon t}[\varepsilon |u_i(t)|^{\vartheta} + \vartheta |u_i(t)|^{\vartheta-1} sign(u_i(t))(-d_i u_i(t) + \sum_{j=1}^{r} a_{ij} f_j^{\varepsilon}(u_j(t)) + \sum_{j=1}^{r} b_{ij} g_j^{\varepsilon}(u_j(t-\tau_{ij}(t)))\\
&\quad + \sum_{j=1}^{r} c_{ij} \int_{-\infty}^{t} K_{ij}(t-s) h_j^{\varepsilon}(u_j(s)) ds)] + e^{\varepsilon t}\left[\sum_{j=1}^{r} |b_{ij}|^{h_{ij}} |\beta_j^{+}|^{j_{ij}} |u_j(t)|^{\vartheta} e^{\varepsilon\tau_{ij}(t)} - \sum_{j=1}^{r} |b_{ij}|^{h_{ij}} |\beta_j^{+}|^{j_{ij}} |u_j(t-\tau_{ij})|^{\vartheta} \right]\\
&\quad + e^{\varepsilon t}\left[\sum_{j=1}^{r} |c_{ij}|^{l_{ij}} |\gamma_j^{+}|^{p_{ij}} |g_j(u_j(t))|^{\vartheta} e^{\varepsilon\tau_{ij}(t)} - \sum_{j=1}^{r} |c_{ij}|^{l_{ij}} |\gamma_j^{+}|^{p_{ij}} |g_j(u_j(t-\tau_{ij}))|^{\vartheta} \right] + \sum_{j=1}^{r} |c_{ij}|^{l_{ij}} |\gamma_j^{+}|^{p_{ij}} |\beta_j^{+}|^{\vartheta} e^{\varepsilon t} |u_j(t-\tau_{ij})|^{\vartheta} \Big\}\\
&\quad + e^{\varepsilon t} \sum_{i=r+1}^{n} \varpi_i \left\{ \sum_{j=r+1}^{n} \left(\vartheta |u_i(t)|^{\vartheta-1} k_i(\bar{u}_r(t)) + \vartheta \sum_{j=r+1}^{n} |c_{ij}| |u_i(t)|^{\vartheta-1} |u_j(t)| \right) + \varepsilon \sum_{j=r+1}^{n} |u_i(t)|^{\vartheta} \right\}
\end{aligned}
$$

By

$$
\begin{aligned}
&\vartheta \sum_{j=r+1}^{n} |c_{ij}| |u_i(t)|^{\vartheta-1} |u_j(t)| \le \vartheta \sum_{j=r+1}^{n} [|c_{ij}|^{\frac{\vartheta-\chi_{ij}}{\vartheta-1}} |u_i(t)|^{\vartheta}]^{\frac{\vartheta-1}{\vartheta}} [|c_{ij}|^{\chi_{ij}} |u_j(t)|^{\vartheta}]^{\frac{1}{\vartheta}}\\
&\le \vartheta \sum_{j=r+1}^{n} \{ \frac{\vartheta-1}{\vartheta} [|c_{ij}|^{\frac{\vartheta-\chi_{ij}}{\vartheta-1}} |u_i(t)|^{\vartheta}]^{\frac{\vartheta-1}{\vartheta}\cdot\frac{\vartheta}{\vartheta-1}} + \frac{1}{\vartheta} [|c_{ij}|^{\chi_{ij}} |u_j(t)|^{\vartheta}]^{\frac{1}{\vartheta}\cdot\vartheta} \}
\end{aligned}
\tag{4}
$$

and

$$
\begin{aligned}
&\vartheta \sum_{j=r+1}^{n} |k_i| |u_i(t)|^{\vartheta-1} \le \vartheta \sum_{j=r+1}^{n} [|k_i|^{\frac{\vartheta-\upsilon_{ij}}{\vartheta-1}} |u_i(t)|^{\vartheta}]^{\frac{\vartheta-1}{\vartheta}} [|k_i|^{\upsilon_{ij}} |u_j(t)|^{\vartheta}]^{\frac{1}{\vartheta}}\\
&\le \vartheta \sum_{j=r+1}^{n} \{ \frac{\vartheta-1}{\vartheta} [|k_i|^{\frac{\vartheta-\upsilon_{ij}}{\vartheta-1}} |u_i(t)|^{\vartheta}]^{\frac{\vartheta-1}{\vartheta}\cdot\frac{\vartheta}{\vartheta-1}} + \frac{1}{\vartheta} [|k_i|^{\upsilon_{ij}} |u_j(t)|^{\vartheta}]^{\frac{1}{\vartheta}\cdot\vartheta} \}
\end{aligned}
\tag{5}
$$

Then, by (4) and (5), we can obtain

$$
\begin{aligned}
&D(u,t)\\
&\le e^{\varepsilon t}\sum_{i=1}^{r}\omega_i\left\{(\varepsilon-\vartheta d_i)|u_i(t)|^{\vartheta}+\vartheta\sum_{j=1}^{r}|a_{ij}|\,|\alpha_j^{+}|\,|u_i(t)|^{\vartheta-1}|u_j(t)|+\vartheta\sum_{j=1}^{r}|b_{ij}|\,|\beta_j^{+}|\,|u_i(t)|^{\vartheta-1}|u_j(t-\tau_{ij}(t))|\right.\\
&\quad+\vartheta\sum_{j=1}^{r}|c_{ij}|\,|\gamma_j^{+}|\,|u_i(t)|^{\vartheta-1}|u_j(t)|\int_0^{+\infty}K_{ij}(s)ds\Big]+\left[\sum_{j=1}^{r}|b_{ij}|^{h_{ij}}|\beta_j^{+}|^{j_{ij}}|u_j(t)|^{\vartheta}e^{\varepsilon\tau_{ij}(t)}-\sum_{j=1}^{r}|b_{ij}|^{h_{ij}}|\beta_j^{+}|^{j_{ij}}|u_j(t-\tau_{ij})|^{\vartheta}\right]\Bigg\}\\
&\quad+e^{\varepsilon t}\sum_{i=r+1}^{n}\varpi_i\left\{\vartheta\sum_{j=r+1}^{n}\{\frac{\vartheta-1}{\vartheta}[|k_i|^{\frac{\vartheta-\upsilon_{ij}}{\vartheta-1}}|u_i(t)|^{\vartheta}]+\frac{1}{\vartheta}[|k_i|^{\upsilon_{ij}}|u_j(t)|^{\vartheta}]\}(\bar{u}_r(t))\right.\\
&\quad+\vartheta(n-r)\sum_{j=r+1}^{n}\{\frac{\vartheta-1}{\vartheta}[|c_{ij}|^{\frac{\vartheta-\chi_{ij}}{\vartheta-1}}|u_i(t)|^{\vartheta}]+\frac{1}{\vartheta}[|c_{ij}|^{\chi_{ij}}|u_j(t)|^{\vartheta}]\}+\varepsilon\sum_{j=r+1}^{n}|u_i(t)|^{\vartheta}\Bigg\}\\
&\le e^{\varepsilon t}\sum_{i=1}^{r}\omega_i\left\{(\varepsilon-\vartheta d_i)|u_i(t)|^{\vartheta}+\vartheta\sum_{j=1}^{r}|a_{ij}|\,|\alpha_j^{+}|\,|u_i(t)|^{\vartheta-1}|u_j(t)|+\vartheta\sum_{j=1}^{r}|b_{ij}|\,|\beta_j^{+}|\,|u_i(t)|^{\vartheta-1}|u_j(t-\tau_{ij}(t))|\right.\\
&\quad+\vartheta\sum_{j=1}^{r}|c_{ij}|\,|\gamma_j^{+}|\,|u_i(t)|^{\vartheta-1}|u_j(t)|\Big]+\left[\sum_{j=1}^{r}|b_{ij}|^{h_{ij}}|\beta_j^{+}|^{j_{ij}}|u_j(t)|^{\vartheta}e^{\varepsilon\tau_{ij}(t)}-\sum_{j=1}^{r}|b_{ij}|^{h_{ij}}|\beta_j^{+}|^{j_{ij}}|u_j(t-\tau_{ij})|^{\vartheta}\right]\Bigg\}\\
&\quad+e^{\varepsilon t}\sum_{i=r+1}^{n}\varpi_i\left\{\sum_{j=r+1}^{n}\{(\vartheta-1)[|k_i|^{\frac{\vartheta-\upsilon_{ij}}{\vartheta-1}}|u_i(t)|^{\vartheta}]+[|k_i|^{\upsilon_{ij}}|u_j(t)|^{\vartheta}]\}(\bar{u}_r(t))\right.\\
&\quad+(n-r)\sum_{j=r+1}^{n}\{(\vartheta-1)[|c_{ij}|^{\frac{\vartheta-\chi_{ij}}{\vartheta-1}}|u_i(t)|^{\vartheta}]+[|c_{ij}|^{\chi_{ij}}|u_j(t)|^{\vartheta}]\}+\varepsilon\sum_{j=r+1}^{n}|u_i(t)|^{\vartheta}\Bigg\}
\end{aligned}
\tag{6}
$$

We are divided into two kinds of discussion

1. when $\vartheta>1$, by Young inequality

$$
\begin{aligned}
&\vartheta\sum_{j=1}^{r}|a_{ij}|\,|\alpha_j^{+}|\,|u_i(t)|^{\vartheta-1}|u_j(t)|\\
&=\vartheta\sum_{j=1}^{r}[|a_{ij}|^{\frac{\vartheta-q_{ij}}{\vartheta-1}}|\alpha_j^{+}|^{\frac{\vartheta-n_{ij}}{\vartheta-1}}|u_i(t)|^{\vartheta}]^{\frac{\vartheta-1}{\vartheta}}[|a_{ij}|^{q_{ij}}|\alpha_j^{+}|^{n_{ij}}|u_j(t)|^{\vartheta}]^{\frac{1}{\vartheta}}\\
&\le\vartheta\sum_{j=1}^{r}[|a_{ij}|^{\frac{\vartheta-q_{ij}}{\vartheta-1}}|\alpha_j^{+}|^{\frac{\vartheta-n_{ij}}{\vartheta-1}}|u_i(t)|^{\vartheta}]^{\frac{\vartheta-1}{\vartheta}}[|a_{ij}|^{q_{ij}}|\alpha_j^{+}|^{n_{ij}}|u_j(t)|^{\vartheta}]^{\frac{1}{\vartheta}}\\
&\le(\vartheta-1)\sum_{j=1}^{r}[|a_{ij}|^{\frac{\vartheta-q_{ij}}{\vartheta-1}}|\alpha_j^{+}|^{\frac{\vartheta-n_{ij}}{\vartheta-1}}|u_i(t)|^{\vartheta}]+\sum_{j=1}^{r}[|a_{ij}|^{q_{ij}}|\alpha_j^{+}|^{n_{ij}}|u_j(t)|^{\vartheta}]
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
&\vartheta\sum_{j=1}^{r}|c_{ij}|\,|\gamma_j^{+}|\,|u_i(t)|^{\vartheta-1}|u_j(t)|\\
&=\vartheta\sum_{j=1}^{r}[|c_{ij}|^{\frac{\vartheta-l_{ij}}{\vartheta-1}}|\gamma_j^{+}|^{\frac{\vartheta-p_{ij}}{\vartheta-1}}|u_i(t)|^{\vartheta}]^{\frac{\vartheta-1}{\vartheta}}[|c_{ij}|^{l_{ij}}|\gamma_j^{+}|^{p_{ij}}|u_j(t)|^{\vartheta}]^{\frac{1}{\vartheta}}\\
&\le\vartheta\sum_{j=1}^{r}[|c_{ij}|^{\frac{\vartheta-l_{ij}}{\vartheta-1}}|\gamma_j^{+}|^{\frac{\vartheta-p_{ij}}{\vartheta-1}}|u_i(t)|^{\vartheta}]^{\frac{\vartheta-1}{\vartheta}}[|c_{ij}|^{l_{ij}}|\gamma_j^{+}|^{p_{ij}}|u_j(t)|^{\vartheta}]^{\frac{1}{\vartheta}}\\
&\le(\vartheta-1)\sum_{j=1}^{r}[|b_{ij}|^{\frac{\vartheta-h_{ij}}{\vartheta-1}}|\beta_j^{+}|^{\frac{\vartheta-j_{ij}}{\vartheta-1}}|u_i(t)|^{\vartheta}]+\sum_{j=1}^{r}[|b_{ij}|^{h_{ij}}|\beta_j^{+}|^{j_{ij}}|u_j(t-\tau_{ij}(t))|^{\vartheta}]
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
&\vartheta\sum_{j=1}^{r}\left|c_{ij}\right|\left|\gamma_j^{+}\right|\left|u_i(t)\right|^{\vartheta-1}\left|u_j(t)\right| \\
&=\vartheta\sum_{j=1}^{r}[\left|c_{ij}\right|^{\frac{\vartheta-l_{ij}}{\vartheta-1}}\left|\gamma_j^{+}\right|^{\frac{\vartheta-p_{ij}}{\vartheta-1}}\left|u_i(t)\right|^{\vartheta}]^{\frac{\vartheta-1}{\vartheta}}[\left|c_{ij}\right|^{l_{ij}}\left|\gamma_j^{+}\right|^{p_{ij}}\left|u_j(t)\right|^{\vartheta}]^{\frac{1}{\vartheta}} \\
&\le\vartheta\sum_{j=1}^{r}[\left|c_{ij}\right|^{\frac{\vartheta-l_{ij}}{\vartheta-1}}\left|\gamma_j^{+}\right|^{\frac{\vartheta-p_{ij}}{\vartheta-1}}\left|u_i(t)\right|^{\vartheta}]^{\frac{\vartheta-1}{\vartheta}}[\left|c_{ij}\right|^{l_{ij}}\left|\gamma_j^{+}\right|^{p_{ij}}\left|u_j(t)\right|^{\vartheta}]^{\frac{1}{\vartheta}} \\
&\le(\vartheta-1)\sum_{j=1}^{r}[\left|c_{ij}\right|^{\frac{\vartheta-l_{ij}}{\vartheta-1}}\left|\gamma_j^{+}\right|^{\frac{\vartheta-p_{ij}}{\vartheta-1}}\left|u_i(t)\right|^{\vartheta}]+\sum_{j=1}^{r}[\left|c_{ij}\right|^{l_{ij}}\left|\gamma_j^{+}\right|^{p_{ij}}\left|u_j(t)\right|^{\vartheta}]
\end{aligned}
\tag{9}
$$

By (6) (7) (8) and (9), we can get as follows

$$
\begin{aligned}
&D^{+}V(u,t) \\
&\le e^{\varepsilon t}\sum_{i=1}^{r}\omega_i\Big\{(\varepsilon-\vartheta d_i)\left|u_i(t)\right|^{\vartheta}+(\vartheta-1)\sum_{j=1}^{r}\left|a_{ij}\right|^{\frac{\vartheta-q_{ij}}{\vartheta-1}}\left|\alpha_j^{+}\right|^{\frac{\vartheta-n_{ij}}{\vartheta-1}}\left|u_i(t)\right|^{\vartheta}+\sum_{j=1}^{r}\left|a_{ij}\right|^{q_{ij}}\left|\alpha_j^{+}\right|^{n_{ij}}\left|u_j(t)\right|^{\vartheta} \\
&\quad+(\vartheta-1)\sum_{j=1}^{r}\left|b_{ij}\right|^{\frac{\vartheta-h_{ij}}{\vartheta-1}}\left|\beta_j^{+}\right|^{\frac{\vartheta-j_{ij}}{\vartheta-1}}\left|u_i(t)\right|^{\vartheta}+\sum_{j=1}^{r}\left|b_{ij}\right|^{h_{ij}}\left|\beta_j^{+}\right|^{j_{ij}}\left|u_j(t-\tau_{ij}(t))\right|^{\vartheta} \\
&\quad+e^{\varepsilon t}\sum_{i=r+1}^{n}\varpi_i\Bigg\{\sum_{j=r+1}^{n}\{(\vartheta-1)[\left|k_i\right|^{\frac{\vartheta-\upsilon_{ij}}{\vartheta-1}}\left|u_i(t)\right|^{\vartheta}]+[\left|k_i\right|^{\upsilon_{ij}}\left|u_j(t)\right|^{\vartheta}]\}(\overline{u}_r(t)) \\
&\quad+(n-r)\sum_{j=r+1}^{n}\{(\vartheta-1)[\left|c_{ij}\right|^{\frac{\vartheta-\chi_{ij}}{\vartheta-1}}\left|u_i(t)\right|^{\vartheta}]+[\left|c_{ij}\right|^{\chi_{ij}}\left|u_j(t)\right|^{\vartheta}]\}+\varepsilon\sum_{j=r+1}^{n}\left|u_i(t)\right|^{\vartheta}\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
&\le e^{\varepsilon t}\sum_{i=1}^{r}\omega_i\Big\{(\varepsilon-\vartheta d_i)+(\vartheta-1)\sum_{j=1}^{r}\left|a_{ij}\right|^{\frac{\vartheta-q_{ij}}{\vartheta-1}}\left|\alpha_j^{+}\right|^{\frac{\vartheta-n_{ij}}{\vartheta-1}}+\sum_{j=1}^{r}\left|a_{ij}\right|^{q_{ij}}\left|\alpha_j^{+}\right|^{n_{ij}}+(\vartheta-1)\sum_{j=1}^{r}\left|b_{ij}\right|^{\frac{\vartheta-h_{ij}}{\vartheta-1}}\left|\beta_j^{+}\right|^{\frac{\vartheta-j_{ij}}{\vartheta-1}} \\
&\quad+(\vartheta-1)\sum_{j=1}^{r}\left|c_{ij}\right|^{\frac{\vartheta-l_{ij}}{\vartheta-1}}\left|\gamma_j^{+}\right|^{\frac{\vartheta-p_{ij}}{\vartheta-1}}+\sum_{j=1}^{r}\left|c_{ij}\right|^{l_{ij}}\left|\gamma_j^{+}\right|^{p_{ij}}+\sum_{j=1}^{r}\left|b_{ij}\right|^{h_{ij}}\left|\beta_j^{+}\right|^{j_{ij}}e^{\varepsilon\rho}+\sum_{j=1}^{r}\left|c_{ij}\right|^{l_{ij}}\left|\gamma_j^{+}\right|^{p_{ij}}\left|\beta_j^{+}\right|^{\vartheta}e^{\varepsilon\rho} \\
&\quad+\sum_{j=1}^{r}\left|c_{ij}\right|^{l_{ij}}\left|\gamma_j^{+}\right|^{p_{ij}}\left|\beta_j^{+}\right|^{\vartheta}\Big\}\left|u_i(t)\right|^{\vartheta}+e^{\varepsilon t}\sum_{i=r+1}^{n}\varpi_i\Bigg\{\sum_{j=r+1}^{n}[(\vartheta-1)\left|k_i\right|^{\frac{\vartheta-\upsilon_{ij}}{\vartheta-1}}+\left|k_i\right|^{\upsilon_{ij}}]\mu_1e^{-\delta_1t}(\underbrace{1,1,\ldots,1}_{r})^{T} \\
&\quad+(n-r)\sum_{j=r+1}^{n}[(\vartheta-1)\left|c_{ij}\right|^{\frac{\vartheta-\chi_{ij}}{\vartheta-1}}+\left|c_{ij}\right|^{\chi_{ij}}]+\varepsilon(n-r)\Bigg\}\left|u_i(t)\right|^{\vartheta}<0
\end{aligned}
$$

where δ_1, μ_1 are certain constants.

2, when $\vartheta=1$, we must let

$$
\begin{aligned}
&q_{ij}=n_{ij}=h_{ij}=j_{ij}=l_{ij}=p_{ij}=q_{ji}=n_{ji}=h_{ji}=j_{ji}=l_{ji}=p_{ji}=1\,(i,j=1,2,\ldots,r), \\
&\upsilon_{ij}=\upsilon_{ji}=\chi_{ij}=\chi_{ji}=1\,(i,j=r+1,\ldots,n)
\end{aligned}
$$

By (6)

$$
\begin{aligned}
&D^{+}V(u,t)\\
&\le e^{\varepsilon t}\sum_{i=1}^{r}\omega_i\left[(\varepsilon-\vartheta d_i)+\sum_{j=1}^{r}\left|a_{ij}\right|\left|\alpha_j^{+}\right|+\sum_{j=1}^{r}\left|c_{ij}\right|\left|\gamma_j^{+}\right|+\sum_{j=1}^{r}\left|c_{ij}\right|\left|\gamma_j^{+}\right|\left|\beta_j^{+}\right|+\sum_{j=1}^{r}\left|b_{ij}\right|\left|\beta_j^{+}\right|e^{\varepsilon\tau_{ij}(t)}\right]\left|u_i(t)\right|\\
&+e^{\varepsilon t}\sum_{i=r+1}^{n}\varpi_i\left\{\sum_{j=r+1}^{n}\left|k_i\right|^{\upsilon_{ij}}\mu_1 e^{-\delta_1 t}\underbrace{(1,1,...,1)}_{r}^{T}+(n-r)\sum_{j=r+1}^{n}\left|c_{ij}\right|^{\chi_{ij}}+\varepsilon(n-r)\right\}\left|u_i(t)\right|<0
\end{aligned}
$$

where δ_1, μ_1 are certain constants, thus, we can learn that when ϑ =1, the conclusions are valid.

4. An example

Consider the following cellular neural network model

$$
\begin{cases}
\dfrac{dx_1(t)}{dt}=-5x_1(t)+\sum_{j=1}^{2}a_{1j}f_j(x_j(t))+\sum_{j=1}^{2}b_{1j}g_j(x_j(t-\tau_{ij}(t)))+0.1\int_{-\infty}^{t}e^{-(t-s)}\tanh(x_1(s))ds\\
\qquad -0.3\int_{-\infty}^{t}2e^{-(t-s)}\tanh(x_2(s))ds+2\\
\dfrac{dx_2(t)}{dt}=-4x_2(t)+\sum_{j=1}^{2}a_{2j}f_j(x_j(t))+\sum_{j=1}^{2}b_{2j}g_j(x_j(t-\tau_{ij}(t)))-0.1\int_{-\infty}^{t}2e^{-(t-s)}\tanh(x_1(s))ds\\
\qquad +0.5\int_{-\infty}^{t}e^{-(t-s)}\tanh(x_2(s))ds+3\\
\dfrac{dx_3(t)}{dt}=k_3(\overline{x}_2(t))+c_{33}x_3(t)
\end{cases}
$$

where $K=\left\{k_{ij}\middle|c_{ij}\right\}_{2\times 2}$, $\int_0^{\infty}e^{\lambda t}K_{ij}(t)dt=k_{ij}<\infty$, $c_{33}=-1, k_3(\overline{x}_2(t))=x_1(t)$, λ is exponential convergence rate estimate.

$$A=\begin{bmatrix}1.001 & 0.801\\ 0 & -1.201\end{bmatrix},\ B=\begin{bmatrix}-2.301 & 1.720\\ 1.102 & 0\end{bmatrix}$$

Activation function as follows

$f_j(x_j(t))=g_j(x_j(t))=\frac{1}{2}(|x+1|-|x-1|)$, $(j=1,2)$, so $\left|\alpha_j^{+}\right|=\left|\beta_j^{+}\right|=\left|\gamma_j^{+}\right|=1$,

If we let

$\omega_i=q_{ij}=n_{ij}=h_{ij}=j_{ij}=l_{ij}=p_{ij}=q_{ji}=n_{ji}=h_{ji}=j_{ji}=l_{ji}=p_{ji}=-\varpi_i$

$=\mu_1=\upsilon_{ij}=\upsilon_{ji}=\chi_{ij}=\chi_{ji}=1, \varepsilon=0, \vartheta=2$

We can easily obtain

$$-\omega_i\vartheta d_i+\sum_{j=1}^{n}\omega_i\left|a_{ij}\right|\left|\alpha_j^{+}\right|+\sum_{j=1}^{n}\omega_i\left|a_{ij}\right|\left|\alpha_j^{+}\right|+\sum_{j=1}^{n}\omega_i\left|b_{ij}\right|\left|\beta_j^{+}\right|+\sum_{j=1}^{n}\omega_i\left|c_{ij}\right|\left|\gamma_j^{+}\right|+\sum_{j=1}^{n}\omega_i\left|c_{ij}\right|\left|\gamma_j^{+}\right|+\sum_{j=1}^{n}\omega_i\left|b_{ij}\right|\left|\beta_j^{+}\right|<0,$$

$$\varpi_i\sum_{j=r+1}^{n}[(\vartheta-1)\left|k_i\right|^{\frac{\vartheta-\upsilon_{ij}}{\vartheta-1}}+\left|k_i\right|^{\upsilon_{ij}}]\mu_1<0,\ \varpi_i(n-r)\sum_{j=r+1}^{n}[(\vartheta-1)\left|c_{ij}\right|^{\frac{\vartheta-\chi_{ij}}{\vartheta-1}}+\left|c_{ij}\right|^{\chi_{ij}}]+\varepsilon(n-r)<0$$

Because the exponential convergence rate and specific nuclear function are not known in advance, so we can only prove the conclusion's correctness in mathematics.

5. Conclusion

A new sufficient condition is derived to guarantee the global exponential stability of the equilibrium point for multi-delay and distributed delay cellular neural network. To the best of our knowledge, compared with traditional methods, our approach is effective.

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